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DIFFRACTION OF ACOUSTIC WAVES ON THE LEADING EDGE OF A FLAT PLATE IN A SUPERSONIC FLOW

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A scheme is proposed for calculating the intensity of the acoustic wave field generated by diffraction of a beam of acoustic waves on a sharp leading edge of a flat plate in a supersonic flow. This wave field is shown to be a functional of the mass-flow amplitude distribution in the acoustic field at the level of the plate surface upstream of the latter. This distribution can be found on the basis of measurements. The discontinuity of the normal-to-plate component of the velocity perturbation on the plate edge plays an important role in determining mass-flow fluctuations along the plate. At large distances from the leading edge of the plate, where the diffraction wave on the boundary-layer edge degenerates into longitudinal acoustic waves, the amplitude of mass-flow fluctuations decreases with increasing distance from the leading edge and depends on wave orientation.

Key words: *boundary layer, hydrodynamic stability, receptivity, laminar–turbulent transition, aeroacoustics.*

1. The problem of diffraction of acoustic waves on bodies immersed into a supersonic flow is considered in the present paper in view of studying receptivity of the near-wall flow (e.g., boundary-layer flow) to acoustic waves and the problem of controlling the transition of the laminar flow to the turbulent state. The degree of influence of acoustic waves on the transition is well known (see, e.g., [1, 2]). It was experimentally found [3] that the most pronounced effect of acoustic waves is observed in the vicinity of the leading edge. Apparently, the influence of acoustic waves on generation of oscillations in the boundary layer from the viewpoint of interaction of the diffraction acoustic field with the near-wall viscous flow was first considered in [4]. To verify the conclusions of Fedorov and Khokhlov [4], Semionov et al. [5] performed experiments where the plate experienced the action of sound from below, and the disturbances were measured in the boundary layer on the upper side shadowed by the acoustic field. Thus, the interaction could proceed with the diffraction wave only. At large distances from the leading edge of the plate, the diffraction wave on the boundary-layer edge degenerates into longitudinal acoustic waves whose fronts can be inclined at different angles to the leading edge of the plate [6]. The intensity of the diffraction wave was low and was not measured in the experiments [5]. At the same time, it was shown [6] that longitudinal acoustic waves can excite high-intensity oscillations in the boundary layer. As a result, low-intensity diffraction waves can generate noticeable oscillations of the flow inside the boundary layer. This circumstance can be responsible for the significant effect of the acoustic field in the vicinity of the leading edge on boundary-layer oscillations at large distances from the leading edge.

As was already noted, it is impossible to identify and measure the amplitude of the diffraction wave in experiments, and it is assumed to be given in computations [6]. Therefore, it does not seem possible at the moment to directly compare the theory and experimental data.

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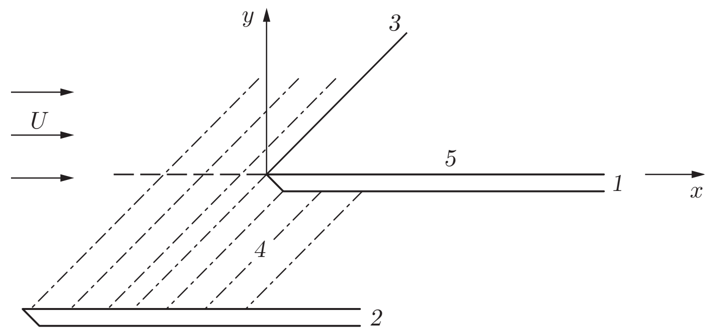

Fig. 1. Layout of the experiment: 1) basic plate; 2) source of sound; 3) Mach line; 4) acoustic field; 5) diffraction zone.

A scheme for calculating the diffraction-wave intensity on the basis of mass-flow fluctuations in the acoustic field at the level of the upper surface of the plate upstream of the latter is proposed in the present paper.

2. Figure 1 shows the schematic of acoustic irradiation of a flat plate in a supersonic flow. The origin $x = 0$ coincides with the leading edge of the plate. The z axis is directed along the leading edge perpendicular to the plane (x, y). The acoustic beam is constricted in the z direction, i.e., its intensity rapidly decreases as $|z| \to \infty$. The problem includes determining of diffraction-wave parameters on the basis of the distribution of disturbances in the region $x < 0$. As was noted above, the problem was mainly posed to compare the experimental [5] and theoretical [6] results. Note, the experiments [5] involved measurements of mass-flow disturbances at a given frequency in the region $x < 0$ for $y = 0$. The measured results were subjected to the Fourier transform with respect to time t and coordinate z. Thus, the experimental data can yield the distribution $\bar{m}_1(\omega, \beta, x, y = 0)$ for $x < 0$, where ω is the frequency parameter and β is the wavenumber in the z direction.

The equations of the vortex-free gas dynamics for disturbances can be written as follows:

$$\frac{du}{dt} = -\frac{1}{\gamma \mathrm{M}^2}\frac{\partial \pi}{\partial x}, \qquad \frac{dv}{dt} = -\frac{1}{\gamma \mathrm{M}^2}\frac{\partial \pi}{\partial y}, \qquad \frac{dw}{dt} = -\frac{1}{\gamma \mathrm{M}^2}\frac{\partial \pi}{\partial z},$$

$$\frac{dr}{dt} = -\Big(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\Big), \qquad \frac{d\theta}{dt} = (1-\gamma)\Big(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\Big), \tag{1}$$

$$\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}, \qquad \frac{\partial u}{\partial z} = \frac{\partial w}{\partial x}, \qquad \frac{\partial v}{\partial z} = \frac{\partial w}{\partial y}, \qquad m_1 = r + u, \qquad \pi = r + \theta.$$

Here, u, v, and w are the velocity disturbances, π is the pressure, θ is the temperature, r is the density, M is the free-stream Mach number, $d/dt = \partial/\partial t + \partial/\partial x$, $m_1(t, z, x, y)$ is the mass flow in the x direction, and γ is the ratio of specific heats. The last relation in (1) was derived from the equation of state. All parameters of the flow and acoustic wave are normalized to their free-stream values. In particular, the free-stream velocity is assumed to be equal to unity. The characteristic linear scale L is not defined in detail at the moment, and the characteristic time can be assumed to be $t_1 = U/L$. It follows from the above-given relations that the mass flow satisfies the wave equation

$$\mathrm{M}^2\frac{d^2 m_1}{dt^2} = \frac{\partial^2 m_1}{\partial x^2} + \frac{\partial^2 m_1}{\partial y^2} + \frac{\partial^2 m_1}{\partial z^2}.$$

Using the direct and inverse Fourier transforms, we find

$$\bar{m}_1(\omega, \beta, x, y) = \frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}\frac{2}{T}\Big(\int\limits_0^T m_1(t, z, x, y)\exp(-i\beta z + i\omega t)\Big)\,dt\,dz,$$

$$m_1(t, z, x, y) = \frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty}\bar{m}_1(\omega, \beta, x, y)\exp(i\beta z - i\omega t)\,d\omega\,d\beta. \tag{2}$$

Substituting the last expression into the wave equation, we obtain

$$\mathrm{M}^2\Big(-i\omega+\frac{\partial}{\partial x}\Big)^2\bar{m}_1=-\beta^2\bar{m}_1+\frac{\partial^2\bar{m}_1}{\partial x^2}+\frac{\partial^2\bar{m}_1}{\partial y^2}. \tag{3}$$

Let us show that the mass flow in the region $x>0$ for $y=0$ can be determined from its gradient:

$$\bar{m}_1(x,y=0)=-\frac{1}{\sqrt{\mathrm{M}^2-1}}\int\limits_0^\infty\frac{\partial\bar{m}_1(x-\bar{x})}{\partial y}\,J_0(a\bar{x})\exp{(i\alpha_0\bar{x})}\,d\bar{x}. \tag{4}$$

Here, J_0 is the zeroth-order Bessel function, $\alpha_0=\mathrm{M}^2\omega/(\mathrm{M}^2-1)$, $a^2=a_0^2+\beta^2/(\mathrm{M}^2-1)$, and $a_0^2=\mathrm{M}^2\omega^2/(\mathrm{M}^2-1)^2$.

Let $\partial\bar{m}_1/\partial y=F(x,y)$ be a known function. We integrate this function in terms of y from zero to infinity, taking into account that m_1 equals zero at infinity. Then, we have $\bar{m}_1(x,0)=-\int\limits_0^\infty F\,dy$. It is known that the acoustic field at an arbitrary point (x,y) can be reconstructed if it is given at the line $y=y_0=0$. In [7], the corresponding relation was obtained in the form

$$F(x,y)=F(x-k,0)\exp{(i\alpha_0k)}-\int\limits_k^\infty\frac{F(x-\bar{x},0)\exp{(i\alpha_0\bar{x})}ak}{\sqrt{\bar{x}^2-k^2}}\,J_1(a\sqrt{\bar{x}^2-k^2})\,d\bar{x},$$

where $k=y\sqrt{\mathrm{M}^2-1}$. Thus, we have

$$m_1(x,0)=m^1+m^2,\qquad m^1=-\frac{1}{\sqrt{\mathrm{M}^2-1}}\int\limits_0^\infty F(x-k,0)\exp{(i\alpha_0k)}\,dk,$$

$$m^2=\frac{1}{\sqrt{\mathrm{M}^2-1}}\int\limits_0^\infty\int\limits_k^\infty\frac{F(x-\bar{x},0)}{\sqrt{\bar{x}^2-k^2}}\exp{(i\alpha_0\bar{x})}akJ_1(a\sqrt{\bar{x}^2-k^2})\,d\bar{x}\,dk.$$

By changing the order of integration, we obtain

$$m^2=-\frac{1}{\sqrt{\mathrm{M}^2-1}}\int\limits_0^\infty F(x-\bar{x},0)\exp{(i\alpha_0\bar{x})}\Big[\int\limits_0^{a\bar{x}}J_1(z)\,dz\Big]\,d\bar{x}$$

$$=-\frac{1}{\sqrt{\mathrm{M}^2-1}}\int\limits_0^\infty F(x-\bar{x},0)\exp{(i\alpha_0\bar{x})}(J_0(a\bar{x})-1)\,d\bar{x}.$$

With allowance for the relation

$$\frac{1}{\sqrt{\mathrm{M}^2-1}}\int\limits_0^\infty F(x-\bar{x},0)\exp{(i\alpha_0\bar{x})}\,d\bar{x}=-m^1,$$

we obtain the expression

$$\bar{m}_1(x,0)=-\frac{1}{\sqrt{\mathrm{M}^2-1}}\int\limits_0^\infty F(x-\bar{x},0)J_0(a\bar{x})\exp{(i\alpha_0\bar{x})}\,d\bar{x},$$

which coincides with Eq. (4).

We find the derivative $\partial\bar{m}_1(x,0)/\partial y$ for $x<0$ under the assumption that the mass-flow distribution in this region is known. In experiments, it is measured directly. For $x>0$, we assume that $\bar{m}_1=0$, which is possible because the disturbances on the plate surface do not affect the region $x<y/\sqrt{\mathrm{M}^2-1}$. In addition, let $\bar{m}_1=0$ in the region $x<-L$. Applying the direct and inverse Fourier transforms, we obtain

$$\bar{\bar{m}}_1(\omega,\beta,\alpha,y)=\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty}^\infty\bar{m}_1(\omega,\beta,x,y)\exp{(-i\alpha x)}\,dx,$$

$$\bar{m}_1(\omega,\beta,x,y) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty+i\delta}^{\infty+i\delta} \bar{\bar{m}}_1(\omega,\beta,\alpha,y)\exp(i\alpha x)\,d\alpha. \tag{5}$$

As $\bar{m}_1 = 0$ for $x < -L$ and $x > 0$, $\bar{\bar{m}}_1(\omega,\beta,\alpha,y)$ is an analytical function in the entire range of α. Substituting the last expression into Eq. (3), we obtain

$$\mathrm{M}^2(-i\omega + i\alpha)^2 \bar{\bar{m}}_1 = -(\beta^2+\alpha^2)\bar{\bar{m}}_1 + \frac{d^2\bar{\bar{m}}_1}{dy^2}. \tag{6}$$

Thus, we have

$$\bar{\bar{m}}_1(\omega,\beta,\alpha,y) = \bar{\bar{m}}_1(\omega,\beta,\alpha,0)\exp(i\lambda),$$

where the expression

$$\lambda = \pm\sqrt{\mathrm{M}^2(\alpha-\omega)^2 - \alpha^2 - \beta^2} = \pm\sqrt{\mathrm{M}^2-1}\sqrt{-a^2+(\Delta\alpha)^2} \qquad (\Delta\alpha = \alpha - \alpha_0)$$

follows from (6), and the sign is chosen to ensure decay as $y \to \infty$ (see [7]). Therefore, we have

$$\bar{m}_1(\omega,\beta,x,y) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty+i\delta}^{\infty+i\delta} \bar{\bar{m}}_1(\omega,\beta,\alpha,0)\exp(i\lambda y + i\alpha x)\,d\alpha.$$

On the basis of the accepted assumptions, we can determine the derivative $\partial\bar{m}_1(x,0)/\partial y$ in the region $x < 0$. We have

$$\frac{\partial\bar{m}_1(\omega,\beta,x,0)}{\partial y} = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty+i\delta}^{\infty+i\delta} i\lambda\bar{\bar{m}}(\omega,\beta,\alpha,0)\exp(i\alpha x)\,d\alpha. \tag{7}$$

Let us show that $\partial m_1/\partial y\Big|_{y=0,x>0} = 0$ on the plate surface. It follows from system (1) that

$$\frac{d}{dt}\Big(\frac{\partial r}{\partial y}\Big) = -\Big(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2}\Big) = -\Delta v, \qquad \frac{d}{dt}\Big(\frac{\partial \pi}{\partial y}\Big) = -\gamma\Delta v.$$

Thus, we have

$$\frac{1}{\gamma}\,\frac{d}{dt}\Big(\frac{\partial\pi}{\partial y}\Big) = \frac{d}{dt}\Big(\frac{\partial r}{\partial y}\Big),$$

$$\frac{d}{dt}\Big(\frac{\partial m_1}{\partial y}\Big) = \frac{d}{dt}\Big(\frac{\partial r}{\partial y}\Big) + \frac{d}{dt}\Big(\frac{\partial u}{\partial y}\Big) = \frac{1}{\gamma}\,\frac{d}{dt}\Big(\frac{\partial\pi}{\partial y}\Big) + \frac{d}{dt}\Big(\frac{\partial v}{\partial x}\Big),$$

$$\frac{d}{dt}\Big(\frac{\partial m_1}{\partial y} - \frac{1}{\gamma}\,\frac{\partial\pi}{\partial y} - \frac{\partial v}{\partial x}\Big) = 0.$$

Since

$$\frac{1}{\gamma}\,\frac{\partial\pi}{\partial y} = -\mathrm{M}^2\,\frac{dv}{dt},$$

we obtain

$$\frac{d}{dt}\Big(\frac{\partial m_1}{\partial y} + \mathrm{M}^2\,\frac{dv}{dt} - \frac{\partial v}{\partial x}\Big) = \frac{d\Psi}{dt} = 0$$

or

$$\frac{\partial\Psi}{\partial t} + \frac{\partial\Psi}{\partial x} = 0.$$

The solution of the last equation is the function $\Psi = \Psi(x-t)$. However, $\Psi(t) = 0$ at $x = -\infty$; hence, $\Psi = 0$ at $t = \infty$ and, as a consequence,

$$\frac{\partial\bar{m}_1}{\partial y} = -(\mathrm{M}^2-1)\,\frac{\partial\bar{v}}{\partial x} - \mathrm{M}^2\,\frac{\partial\bar{v}}{\partial t}. \tag{8}$$

Since the normal velocity on the plate surface equals zero, we have $\partial\bar{m}_1/\partial y\Big|_{y=0} = 0$. Thus, with allowance for Eq. (7), the mass-flow gradient for $y = 0$ is set in the entire range of x except for $x = 0$.

Let us assume, however, that $\partial m/\partial y$ is set along the entire x axis including the point $x = 0$. As is shown below, this assumption allows us to solve the problem being posed.

The normal component of velocity has a discontinuity at $x = 0$; therefore, relation (8) can be taken in the form $\partial\bar{m}_1(x, 0)/\partial y = -(\mathrm{M}^2 - 1)\,\partial\bar{v}/\partial x$. With allowance for $\partial\bar{m}_1(x)/\partial y = 0$ for $x > 0$, equality (4) acquires the form

$$\bar{m}_1(x, y = 0) = \lim_{\varepsilon\to 0}\Big[-\frac{1}{\sqrt{\mathrm{M}^2 - 1}}\int\limits_{x+\varepsilon}^{\infty}\frac{\partial\bar{m}_1(x - \bar{x})}{\partial y}\,J_0(a\bar{x})\exp{(i\alpha_0\bar{x})}\,d\bar{x}$$

$$-(\mathrm{M}^2 - 1)\bar{v}(-\varepsilon, 0)J_0(ax)\exp{(i\alpha_0 x)}\Big]. \tag{9}$$

We substitute Eq. (7) into the first term in the right side of Eq. (9). (Note, the discontinuity of the mass-flow gradient is taken into account by the second term.) As a result, we obtain

$$m^1(\omega, \beta, x, 0) = -\frac{1}{\sqrt{\mathrm{M}^2 - 1}}\frac{1}{\sqrt{2\pi}}\int\limits_{x}^{\infty}\Big[\int\limits_{-\infty+i\delta}^{\infty+i\delta} i\lambda\bar{\bar{m}}\exp{[i\alpha(x - \bar{x})]}\,d\alpha\Big]J_0(a\bar{x})\exp{(i\alpha_0\bar{x})}\,d\bar{x}$$

$$= -\frac{1}{\sqrt{\mathrm{M}^2 - 1}}\frac{1}{\sqrt{2\pi}}\int\limits_{-\infty+i\delta}^{\infty+i\delta}\Big[i\lambda\exp{(i\alpha x)}\bar{\bar{m}}\int\limits_{x}^{\infty}J_0(a\bar{x})\exp{[i(\alpha_0 - \alpha)\bar{x}]}\,d\bar{x}\Big]\,d\alpha.$$

For $\delta < 0$, the integral over $\bar{x}$ exists. Therefore, the integrand is an analytical function of the variable α in the lower half-plane. Closing the integration contour in the range of negative values of δ, we obtain $\bar{m}_1 = 0$. This obviously follows from the analysis of the region of discontinuity of the gradient $\partial m/\partial y$. For the formal acoustic beam, it was *a priori* assumed that $\bar{m}_1(x) = 0$ in the region $x > 0$. It was under this assumption that the transformation $\bar{\bar{m}}(\alpha)$ was obtained. Thus, the mass-flow fluctuations along the plate are completely determined by the discontinuity of the normal component of velocity disturbances at the plate edge. Let us find the relation between the normal component of velocity $\bar{v}(x)$ upstream of the plate edge and the mass-flow distribution in the region $x < 0$. Relation (8) on the line $y = 0$ yields the differential equation

$$\bar{m}^0_y(x) = -(\mathrm{M}^2 - 1)\,\frac{\partial\bar{v}^0}{\partial x} - \mathrm{M}^2\frac{\partial\bar{v}^0}{\partial t}.$$

Applying the direct and inverse Fourier transforms [similar to (2) and (5)] to $\bar{v}^0$, with allowance for Eq. (6), we obtain

$$\bar{\bar{v}}_0(\alpha) = -\frac{1}{\mathrm{M}^2 - 1}\,\frac{\lambda(\alpha)\bar{\bar{m}}(\alpha)}{\alpha - \omega\mathrm{M}^2/(\mathrm{M}^2 - 1)},$$

$$\bar{v}_0(x = 0) = -\frac{1}{\sqrt{2\pi}(\mathrm{M}^2 - 1)}\int\limits_{-\infty+i\delta}^{\infty+i\delta}\frac{\lambda(\alpha)\bar{\bar{m}}(\alpha)\,d\alpha}{\alpha - \omega\mathrm{M}^2/(\mathrm{M}^2 - 1)}.$$

For each (upper or lower) half-plane of α, we can choose the branch of λ satisfying the condition $\mathrm{Im}\,(\lambda) = 0$. In this case, the integrand in each half-plane is analytical. For certainty, we choose the upper half-plane of α ($\delta > 0$). Directing δ to zero and avoiding the singular point, we obtain

$$\bar{v}^0(0) = \sqrt{\pi/2}\,i\lambda(\alpha_0)\bar{\bar{m}}(\alpha_0)/(\mathrm{M}^2 - 1).$$

Note, processing of experimental data often involves the Fourier transform in the form

$$\tilde{m} = \int\limits_{-\infty}^{\infty}\bar{m}_1\exp{(-i\alpha x)}\,dx,$$

i.e., $\tilde{m} = \sqrt{2\pi}\,\bar{\bar{m}}$. In addition, the following condition is valid for disturbances decaying at infinity: $\lambda(\alpha_0) = ia\sqrt{\mathrm{M}^2 - 1}$. Therefore, the dependence of the mass flow on x [see the second term in the right side of Eq. (9)] can be written as

$$\bar{m}_1(x, \beta, \omega) = -(1/2)a\sqrt{\mathrm{M}^2 - 1}\,\tilde{m}(\alpha_0)J_0(ax)\exp(i\alpha_0 x). \tag{10}$$

Taking into account the asymptotic behavior of the Bessel function at large values of x, we convert Eq. (10) to the form

$$\bar{m}_1(x, \beta, \omega) \approx -\frac{1}{2}\sqrt{\mathrm{M}^2 - 1}\,\tilde{m}(\alpha_0)\sqrt{\frac{a}{2\pi x}}\left\{\exp\left[i\left(\alpha_1 x - \frac{\pi}{4}\right)\right] + \exp\left[i\left(\alpha_2 x + \frac{\pi}{4}\right)\right]\right\}, \tag{11}$$

where $\alpha_{1,2} = \alpha_0 \mp a$. Using the expressions for α_0 and a and Eq. (4), we can show that $\alpha_{1,2} = \omega\bar{\mathrm{M}}/(\bar{\mathrm{M}} \mp 1)$, where $\bar{\mathrm{M}} = \mathrm{M}\cos\chi$ and $\chi = \arctan(\beta/\alpha_{1,2})$.

Thus, the intensity of mass-flow fluctuations decreases with increasing distance from the leading edge and can vary depending on orientation of the initial wave, whereas the latter depends on the spectrum of $\tilde{m}(\alpha, \omega, \beta)$ over β and on the magnitude of a, which increases with increasing β. Formulas (10) and (11) establish the relation between the mass flow in the region $x > 0$ with its distribution in the region $x < 0$.

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